Math 335 **Final Exam** **Tuesday, December 17, 2002**

Name: ______________________________

Directions: Be neat. Show work. Use separate paper. Please staple your crib sheet to your exam when you turn it in.

1. (16 points) The second factorial moment for a random variable X is defined by $E\big(X(X-1)\big)$.
   (a) For this part, assume that you do NOT know the mean and variance of a Poisson. Derive the second factorial moment of a Poisson with parameter λ by using the definition of second factorial moment and the pdf for a Poisson.
   (b) Now, do it another way. Use the definition of second factorial moment along with what you know about the mean and variance of a Poisson to derive the second factorial moment.

2. (18 points) Two teams A and N play each other in a series of games. The first team to win 2 games wins the series. The teams play the first game at A's home field, the second at N's home field, and the third (if needed) at A's home field. Suppose games are independent and the probability the home team wins a game is always .6.
   (a) Find the probability A wins the series.
   (b) Find the expected length of the series where length is the number of games the series lasts till conclusion.
   (c) Suppose a series lasts 3 games. What is the probability that A won the series?

3. (12 points) Recall a gamma pdf is of the form $f_X(x) = \dfrac{\lambda^r}{\Gamma(r)}\, y^{r-1}\, e^{-\lambda y},\ y > 0$ where λ and r are positive constants.

   Recall $E(X) = \dfrac{r}{\lambda}$ and $\mathrm{Var}(X) = \dfrac{r}{\lambda^2}$, and M-GF $M_X(t) = \left(\dfrac{1}{1-\frac{t}{\lambda}}\right)^r$, if $t < \lambda$.

   Suppose X_1 is gamma with parameters λ and r_1 and X_2 is an independent gamma pdf with parameters λ and r_2. Prove that $X_1 + X_2$ is also gamma. Make your reasoning clear.

(Please turn over.)

4. (40 points) The cross-sectional area of plastic tubing for use in pulminary resuscitators is normally distributed with $\mu = 12.50$ mm and $\sigma = 0.2$ mm. When the area is less than 12.2 mm or greater than 12.80 mm, the tube does not fit properly.

   (a) Find the probability that a randomly selected tube does not fit properly.

   (b) Suppose a shipment contains 5 tubes. What is the probability that more than 2 tubes will not fit properly?

   (c) Suppose tubes are typically shipped out in boxes of 1000. Find the probability that less than 15% of the tubes in a box will not fit properly.

   (d) Suppose the company decides to ship out in boxes of n tubes. Find the smallest possible n so that there is a 95% chance that the proportion of improperly fitting tubes in the box is less than .15.

5. (10 points) A dashboard warning light is supposed to flash red if a car's oil pressure is too low. On a certain model, the probability of the light flashing when it should is 0.99 or 99%. But 2% of the time it flashes for no apparent reason. If there is a 10% chance that the oil pressure really is low, what is the probability that a driver needs to be concerned if the warning light goes on?

6. (24 points) Consider a random variable X such that the pdf is

$$f_X(x) = \frac{c}{\sqrt{x}}, \quad 0 < x < 1, \quad c \text{ is a constant.}$$

   (a) Find c.

   (b) Find $E(\sqrt{X})$.

   (c) Find $f_Y(y)$ where random variable $Y = \ln X$. Make the support clear.