

Math 335.01

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Signature: \_\_\_\_\_

### Exam III

**Directions:** This is a take-home exam. You may use your textbook, the “Introduction to R” document, the class web page, the R web page, and any of your class notes or homework papers. You may **not** use other books or resources. And you must **work alone**. You should have no conversation of any kind about this exam with anyone, including incidental or trivial comments. (The lone exception would be that you can consult with the course instructor, Tom Moore, should you need clarification of some point.) Your signature above will indicate that you have followed these rules. Submit this signed exam copy to the front of your solutions. Be neat in solving the problems, showing your work and justifying all assertions. I will grade for clarity and neatness as well as correctness. Where possible, give a final numerical answer.

The exam is due at 2:15 p.m., in class, on Monday, December 3, 2007.

1. [20 points] Let  $Y$  be a continuous random variable with pdf given by

$$f_Y(y) = \begin{cases} e^{-(y-\theta)}, & \theta < y \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Derive the moment-generating function for  $Y$ . Show your work.  
(b) What is the domain of the moment-generating function?  
(c) Use (a) and Theorem 3.12.1 to find  $E(Y)$  and  $Var(Y)$ .
2. [10 points] If  $X$  and  $Y$  are independent random variables with moment-generating functions given by  $M_X(t) = e^{-4+4e^t}$  and  $M_Y(t) = (\frac{1}{3} + \frac{2}{3}e^t)^2$ , find  $P(X = 2Y)$ . Explain and **justify** your work clearly.
3. [18 points] Let  $X_1, X_2, \dots, X_9$  be a random sample from a  $N(2, 4)$  distribution. Let  $Y_1, Y_2, Y_3, Y_4$  be a random sample from a  $N(1, 1)$  distribution. Assume the  $X_i$ 's are also independent of the  $Y_i$ 's. Note: The notation  $N(\mu, \sigma^2)$  is for a normal distribution where the first argument is the mean and the second is the **variance**. Find the following, making your work clear.
- (a)  $P(\bar{X} > 1 \text{ and } \bar{Y} > 2)$   
(b)  $P(\bar{X} \geq \bar{Y})$   
(c) The expected value (mean) and variance of the random variable  $3\bar{X} + 2\bar{Y}$ .

**Please Turn Over**

4. [16 points] Ignore leap year for this problem, which is about birthdays. A group of 100 people is chosen at random from the general population. We are interested in the day of the year they were born on, i.e., their “birthday.” (We don’t care what year they were born in.)
- (a) Let  $X_i = 1$  if exactly two people in the group have day  $i$  as their birthday; otherwise  $X_i = 0$ ;  $i = 1, 2, \dots, 365$ . Find  $E(X_i)$ .
  - (a) Find the expected number of days that are birthdays of exactly 2 people.
  - (c) Use an R simulation with (at least) 10,000 iterations to corroborate your answer to (b). Include R code, output, and an appropriate histogram and compare the simulation results to your answer for (b). Note: The `tabulate` function might be useful here. (See the “R functions and code” section of the course web page.)
5. [20 points] A grasshopper is sitting on a straight, East-West curb. Each minute the grasshopper does one of the following at random: it stays put with probability  $1/4$ , it jumps a foot to the East with probability  $1/2$ , or it jumps a foot to the West with probability  $1/4$ . Suppose the grasshopper performs 80 minutes of this process.
- (a) Find the mean and variance of the grasshopper’s location on the curb, relative to its starting position.
  - (b) Find the probability the grasshopper is more than 30 feet to the east of his starting point.
  - (c) Run an R simulation that simulates the 80-minute experiment 10,000 (or more) times. Give the R code, a histogram of results, and report on R output that relates to the answers to (a) and (b). Summarize the results of your simulation, comparing them to your answers to (a) and (b).
6. [12 points] Suppose a certain variety of insect, called a leaf crawler, infects an apple orchard. The number of insects on a randomly inspected leaf is a Poisson random variable with  $\lambda = 1.5$ . Assume for the sake of this exercise that leaves behave as independent random variables and  $\lambda = 1.5$  for all leaves.
- (a) Find the probability that a random leaf has more than 2 leaf crawlers on it.
  - (b) Now, take a random sample of 5 leaves. What is the **exact** probability that a total of 4 or fewer leaf crawlers will be found on the 5 leaves? Justify the use of the probability distribution you use.