Math 335.01

Weekly Problems #5 (Due Friday, October 10, 2008)

1. Let X have the following probability density function:

$$f_X(x) = \begin{cases} 1, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Y = \sqrt{X}$. That is, we generate a number uniformly between 0 and 1, called X, and then we take the square root of this number and call it Y.

   (a) Use R to simulate 10000 instances of Y placed into a vector. Give a histogram of the vector to illustrate that Y does not have a uniform distribution. Note: Use the `probability=T` argument in the `hist` function call.

   (b) Based upon (a), make a guess as to the pdf for Y.

   (c) Now get the theoretical CDF, that is for any y between 0 and 1, compute $F_Y(y) = P(Y \leq y)$.

   (d) Write down the formula for the CDF for Y, giving its value for all values of y.

   (e) Use (d) and Theorem 3.4.1 to find the pdf for Y. The support of the pdf should be clear; that is, make sure you have defined the pdf for all values of y.

2. Problem (1) suggests a general method for finding the pdf of a random variable Y that is a function of another random variable X for which the pdf is known. The idea is to first find the CDF for Y and then use Theorem 3.4.1 to find the pdf, through differentiation. We will call this method the "CDF method."

   (a) Use the CDF method to find the pdf for $Y = X^2$, where X is as given in problem (1).

   (b) Use an R simulation and graph to corroborate the result from (a).