Math 335.01

Weekly Problems #1

Let $A_1, A_2, \ldots, A_n$ be events in a sample space S. Boole's inequality says that

$$P\Big(\bigcup_{i=1}^{n} A_i\Big) \leq \sum_{i=1}^{n} P(A_i)$$

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(a) Prove Boole's inequality for $n = 2$.

(b) Prove Boole's inequality for all n.

(c) (The Fermat-Pascal dice problem) Recall that the Chevalier de Mere originally thought that with 24 rolls of two dice, the chance of at least one "double ace" would be slightly greater than .50, but Fermat and Pascal figured out that 25 rolls were necessary. Tom's on-line R document shows how to simulate this experiment using R and the `replicate` function. Consider and solve the next case up: How many rolls (n) of 3 dice are needed to have a better than .50 chance of tossing a "triple ace?" Answer the question by giving an analytical argument and then produce an R simulation that confirms the answer. On the latter, use 100,000 iterations in your final answer. With your write-up, include your final value of n along with the simulated probability of at least one "triple ace" for that n and the probability for $n - 1$. Also include your R code (which you may just copy by hand into your write-up.)

(d) For the "triple ace" problem in (c), let A_i be the event of a triple ace on the *ith* roll. With this interpretation and using n as obtained in part (c), what does Boole's inequality tell you about the probability of at least one triple ace with n rolls?

Weekly Problems #2

Suppose four old, former college friends are visiting their alma mater one day, but none of them realize this. There are 15 restaurants in town. Assume that the four individuals choose a restaurant at random for lunch at noon.

(a) Find the probability that at least two of them will choose the same restaurant? (In which case, they will have a chance meeting.)

(b) Use an R simulation to estimate the answer to (a) and compare the estimate to the theoretical answer. Note: I found these R functions to be helpful: `sample` (using the `replace=TRUE` argument), `unique` and `length`.

(c) Find an expression for answering the same problem if there are k friends and n restaurants with $k < n$.

(d) Call the answer to (c), $P_{n,k}$. Use (without proof) the approximation $\ln(1+z) \approx z$, for small z, to reexpress $P_{n,k}$. Now assume $n = 365$ and find the value of k for which $P_{n,k} \approx \frac{1}{2}$.