Math 335 Problems **Fall 2008** **T. Moore**

1. If $f_X(x) = 6x(1-x),\ 0 < x < 1$, find $f_Y(y)$, where $Y = 2X - 3$.

2. Suppose that X has the uniform pdf over the interval (a, b). What linear transformation of X represents a random variable having the uniform pdf over $(0, 1)$?

3. Suppose that X has pdf $f_X(x) = e^{-x}$, $x > 0$.
   (a) Find the pdf for $Y = 1/X$.
   (b) Find the pdf for $Y = \ln X$.

4. Find the pdf $f_Y(y)$ of the random variable $Y = \ln X$, where $f_X(x) = xe^{-x^2/2}$, $x \geq 0$.

5. If the random variable X has pdf $f_X(x) = 3x^2$, $0 < x < 1$, and if $Y = 4X^2$, what is $f_Y(y)$?

6. A random variable X has density function $f_X(x) = \frac{3}{8}x^2$ over the interval $0 \leq x \leq 2$ (and 0, elsewhere). Suppose that a circle is generated by a radius whose length is the value of X. Find the density function for Y, the *area* of the circle.

7. Find the pdf of $R = A \cdot \sin\theta$, where A is a constant and θ is uniformly distributed on $(-\pi/2, \pi/2)$. Variables like R arise quite often in the theory of ballistics. If a projectile is fired at an angle α with a velocity v, the distance R that it travels can be expressed as $R = (v^2/g) \cdot \sin 2\alpha$, where g is the gravitational constant.

8. Let X and Y have the bivariate uniform pdf over the unit square. Define $Z = X + Y$. Find $F_Z(z)$. (*Hint:* Consider two cases, $0 < z \leq 1$ and $1 < z < 2$. For each case, draw a diagram showing the appropriate region of integration.)

9. Let X and Y be a continuous random variables for which $f_{X,Y}(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$. Find $f_Z(z)$, where $Z = XY$.

10. Suppose that the random variables X and Y have the joint uniform density over the unit square. Find (a) the cdf and (b) the pdf for $Z = X/Y$.

11. A number is chosen at random from the interval $(0, 3)$. A second number is chosen independently and at random from the interal $(0, 4)$. What is the eightieth percentile of the sum of the two numbers? [By definition, the 80th percentile, z_{80}, is that number for which $P(X + Y \leq z_{80}) = 0.80$.]

12. Suppose X has the uniform pdf over the interval $(-1, 2)$:

$$f_X(x) = \begin{cases} \frac{1}{3}, & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the pdf for Y, where $Y = X^2$.