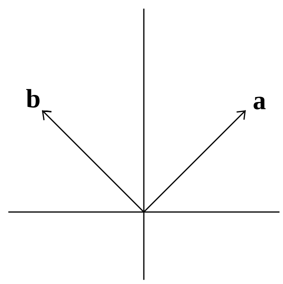
Math 151 Challenge Problems

Homework 2

due: Sept. 16, 2003

1. We call the special vectors $\mathbf{i}$ and $\mathbf{j}$ the *standard basis vectors*, and recall that we can express any vector in terms of these two unit vectors: $\mathbf{v} = \langle -1, 3\rangle = -\mathbf{i} + 3\mathbf{j}$. We can find other sets of vectors that form a basis, which means that we can express any vector (uniquely) in terms of the vectors in the basis.

Let $\mathbf{a} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$ and $\mathbf{b} = \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\rangle$. Note that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors.

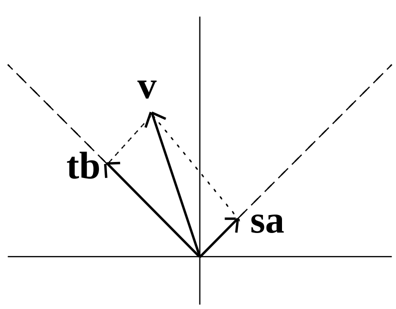


The vectors $\mathbf{a}$ and $\mathbf{b}$ form a basis.

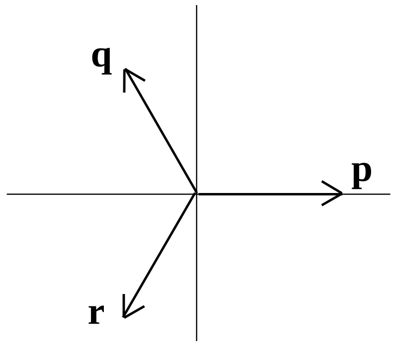
a) Find a simplified expression for the vector projection of a vector $\mathbf{v}$ onto a unit vector unit vector $\mathbf{u}$.

b) Find the vector projection of $\mathbf{v} = \langle -1, 3\rangle$ onto the unit vectors $\mathbf{a}$ and $\mathbf{b}$.

c) Write the vector $\mathbf{v} = \langle -1, 3\rangle$ in terms of $\mathbf{a}$ and $\mathbf{b}$. In other words, find scalar values s and t for which $\mathbf{v} = s\mathbf{a} + t\mathbf{b}$. Hint: See figure below.



2. Let $\mathbf{p} = \langle 1, 0 \rangle$, $\mathbf{q} = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$, and $\mathbf{r} = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle$.



a) Write the vector $\mathbf{v} = \langle -2, 0 \rangle$ in terms of $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$ in two different ways. Because a given vector can have more than one expression, this set of vectors is not called a basis.

b) This set of vectors does have a neat property, though. Verify the following:

$$\begin{aligned} \mathbf{v} = \langle v_1, v_2 \rangle &= \frac{2}{3}[\operatorname{proj}_{\mathbf{p}}\mathbf{v} + \operatorname{proj}_{\mathbf{q}}\mathbf{v} + \operatorname{proj}_{\mathbf{r}}\mathbf{v}] \\ &= \frac{2}{3}[(\mathbf{v}\cdot\mathbf{p})\mathbf{p} + (\mathbf{v}\cdot\mathbf{q})\mathbf{q} + (\mathbf{v}\cdot\mathbf{r})\mathbf{r}] \end{aligned}$$