

Assignment 1

- (1) A rectangular building is being designed to minimize heat loss. The east and west walls lose heat at a rate of 10 units/m² per day, the north and south walls at a rate of 8 units/m² per day, the floor at a rate of 1 unit/m² per day, and the roof at a rate of 5 unit/m² per day. Each wall must be at least 30m long, the height must be at least 4m, and the volume must be exactly 4000 m³.
 - (a) Find the dimensions which minimize heat loss.
 - (b) Could you design a building with even less heat loss if the restrictions on the lengths of the walls were removed?
- (2) A hallway which is 3 metres wide meets a 90 degree corner and continues as a 4 metre wide corridor. Some long heavy pipes need to be carried horizontally around this bend. How long can the pipes be? Express your answer exactly, not numerically.
- (3) The blood vascular system consists of blood vessels (arteries, arterioles, capillaries, and veins) that convey blood from the heart to the organs and back to the heart. This system should work so as to minimize the energy expended by the heart in pumping the blood. In particular, this energy is reduced when the resistance of the blood is lowered. One of Poiseuille's Laws gives the resistance R of the blood as

$$R = c \frac{L}{r^4}$$

where L is the length of the blood vessel, r is the radius, and C is a positive constant determined by the viscosity of the blood. (Poiseuille established this law experimentally, but it also follows by setting up the appropriate integral). The figure shows a main blood vessel with radius r_1 branching at an angle θ into a smaller vessel with radius r_2 .

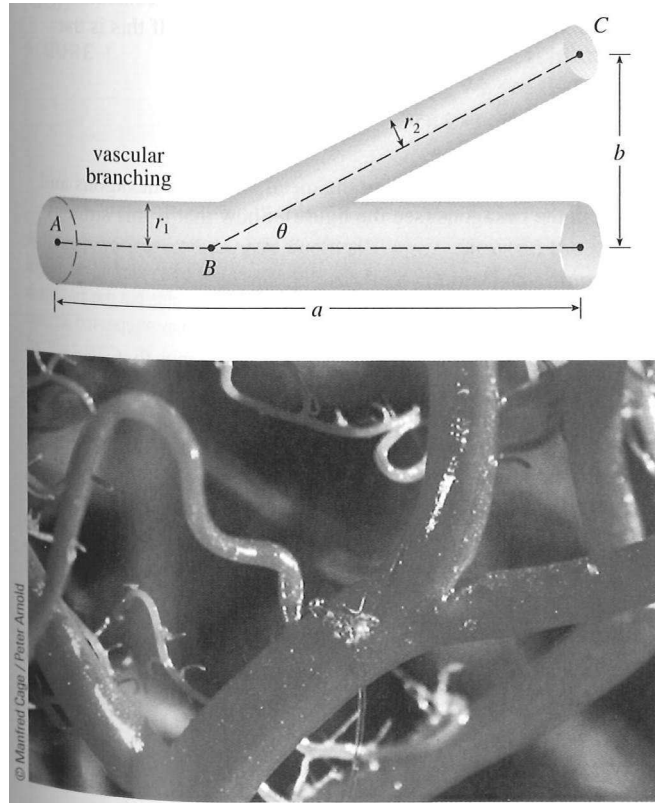


FIGURE 1. Vascular Branching.

- (a) Use Poiseuille's Law to show that the total resistance of the blood along the path ABC is

$$R = C \left(\frac{a - b \cot \theta}{r_1^4} + \frac{b \csc \theta}{r_2^4} \right)$$

where a and b are the distances shown in the figure.

- (b) Prove that the resistance is minimized when

$$\cos \theta = \frac{r_2^4}{r_1^4}$$

- (c) Find the optimal branching angle (to the nearest degree) when the radius of the smaller blood vessel is two-thirds the radius of the larger vessel.